Exercise 24

For the following exercises, determine the domain and range of the quadratic function.

$$f(x) = 2x^2 - 4x + 2$$

Solution

Any value of x can be plugged into a polynomial function, so the domain is

$$\{x \mid -\infty < x < \infty\}.$$

In order to determine the range, first write the quadratic function in vertex form by completing the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Factor the coefficient of x^2.

$$f(x) = 2(x^2 - 2x + 1)$$

Notice that $2B = -2$, which means $B = -1$ and $B^2 = 1$. Add and subtract 1 on the right side and then use the identity to make x appear in only one place rather than two.

$$f(x) = 2[(x^2 - 2x + 1) + 1 - 1]$$
$$= 2[(x + (-1))^2 + 0]$$
$$= 2(x - 1)^2$$

Because the coefficent of the squared term is positive, the parabola opens upward; in other words, the squared term takes on values between zero and infinity. The smallest value of $f(x)$ is 0, and the highest value of $f(x)$ is ∞.

$$\{y \mid 0 \leq y < \infty\}$$